

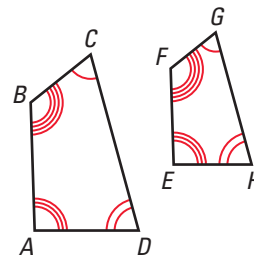
8.3

Similar Polygons

What you should learn**GOAL 1** Identify similar polygons.**GOAL 2** Use similar polygons to solve **real-life** problems, such as making an enlargement similar to an original photo in **Example 3**.**Why** you should learn it▼ To solve **real-life** problems, such as comparing television screen sizes in **Exs. 43 and 44**.**GOAL 1** IDENTIFYING SIMILAR POLYGONS

When there is a correspondence between two polygons such that their corresponding angles are congruent and the lengths of corresponding sides are proportional the two polygons are called **similar polygons**.

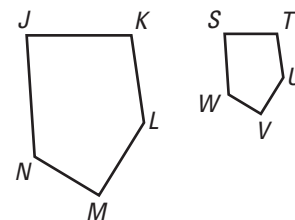
In the diagram, $ABCD$ is similar to $EFGH$. The symbol \sim is used to indicate similarity. So, $ABCD \sim EFGH$.



$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

EXAMPLE 1 Writing Similarity Statements

Pentagons $JKLMN$ and $STUVW$ are similar. List all the pairs of congruent angles. Write the ratios of the corresponding sides in a statement of proportionality.

**SOLUTION**

Because $JKLMN \sim STUVW$, you can write $\angle J \cong \angle S$, $\angle K \cong \angle T$, $\angle L \cong \angle U$, $\angle M \cong \angle V$, and $\angle N \cong \angle W$.

You can write the statement of proportionality as follows:

$$\frac{JK}{ST} = \frac{KL}{TU} = \frac{LM}{UV} = \frac{MN}{VW} = \frac{NJ}{WS}$$

EXAMPLE 2 Comparing Similar Polygons

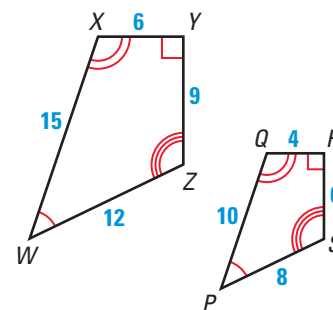
Decide whether the figures are similar. If they are similar, write a similarity statement.

SOLUTION

As shown, the corresponding angles of $WXYZ$ and $PQRS$ are congruent. Also, the corresponding side lengths are proportional.

$$\frac{WX}{PQ} = \frac{15}{10} = \frac{3}{2} \quad \frac{XY}{QR} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{YZ}{RS} = \frac{9}{6} = \frac{3}{2} \quad \frac{ZW}{SP} = \frac{12}{8} = \frac{3}{2}$$



► So, the two figures are similar and you can write $WXYZ \sim PQRS$.

STUDENT HELP**Study Tip**

When you refer to similar polygons, their corresponding vertices must be listed in the same order.

GOAL 2 USING SIMILAR POLYGONS IN REAL LIFE

EXAMPLE 3 Comparing Photographic Enlargements



POSTER DESIGN You have been asked to create a poster to advertise a field trip to see the Liberty Bell. You have a 3.5 inch by 5 inch photo that you want to enlarge. You want the enlargement to be 16 inches wide. How long will it be?

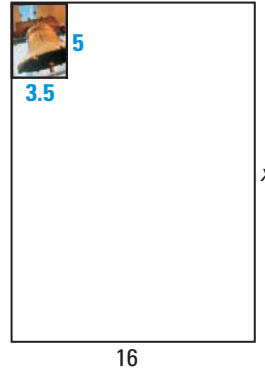
SOLUTION

To find the length of the enlargement, you can compare the enlargement to the original measurements of the photo.

$$\frac{16 \text{ in.}}{3.5 \text{ in.}} = \frac{x \text{ in.}}{5 \text{ in.}}$$

$$x = \frac{16}{3.5} \cdot 5$$

$$x \approx 22.9 \text{ inches}$$



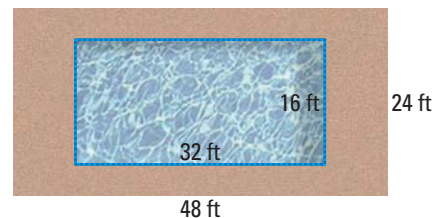
▶ The length of the enlargement will be about 23 inches.

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If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 2 on the previous page, the common ratio of $\frac{3}{2}$ is the scale factor of $WXYZ$ to $PQRS$.

EXAMPLE 4 Using Similar Polygons

The rectangular patio around a pool is similar to the pool as shown. Calculate the scale factor of the patio to the pool, and find the ratio of their perimeters.



SOLUTION

Because the rectangles are similar, the scale factor of the patio to the pool is 48 ft:32 ft, which is 3:2 in simplified form.

The perimeter of the patio is $2(24) + 2(48) = 144$ feet and the perimeter of the pool is $2(16) + 2(32) = 96$ feet. The ratio of the perimeters is $\frac{144}{96}$, or $\frac{3}{2}$.

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Notice in Example 4 that the ratio of the perimeters is the same as the scale factor of the rectangles. This observation is generalized in the following theorem. You are asked to prove Theorem 8.1 for two similar rectangles in Exercise 45.

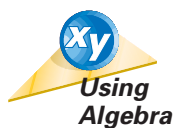
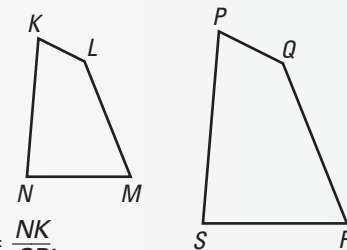
THEOREM

THEOREM 8.1

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KL MN \sim PQRS$, then

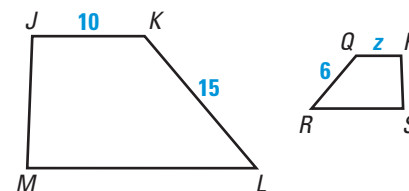
$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}.$$



EXAMPLE 5 Using Similar Polygons

Quadrilateral $JKLM$ is similar to quadrilateral $PQRS$.

Find the value of z .



SOLUTION

Set up a proportion that contains PQ .

$$\frac{KL}{QR} = \frac{JK}{PQ} \quad \text{Write proportion.}$$

$$\frac{15}{6} = \frac{10}{z} \quad \text{Substitute.}$$

$$z = 4 \quad \text{Cross multiply and divide by 15.}$$

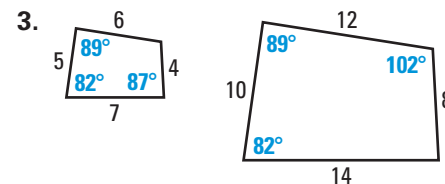
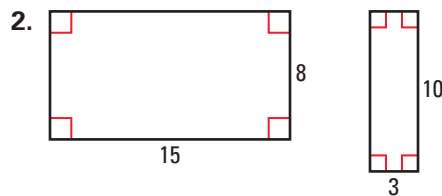
GUIDED PRACTICE

Vocabulary Check ✓

1. If two polygons are similar, must they also be congruent? Explain.

Concept Check ✓

Decide whether the figures are similar. Explain your reasoning.



Skill Check ✓

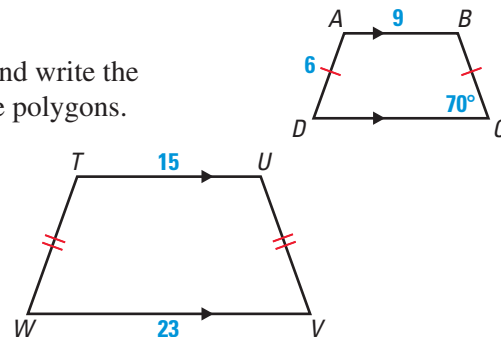
In the diagram, $TUVW \sim ABCD$.

4. List all pairs of congruent angles and write the statement of proportionality for the polygons.

5. Find the scale factor of $TUVW$ to $ABCD$.

6. Find the length of \overline{TW} .

7. Find the measure of $\angle TUV$.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 817.

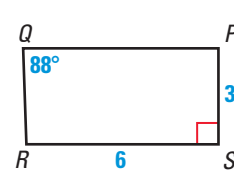
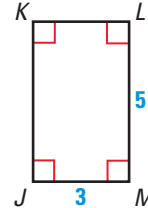
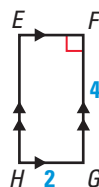
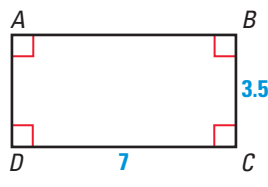
WRITING SIMILARITY STATEMENTS Use the information given to list all pairs of congruent angles and write the statement of proportionality for the figures.

8. $\triangle DEF \sim \triangle PQR$

9. $\square JKLM \sim \square WXYZ$

10. $QRSTU \sim ABCDE$

DETERMINING SIMILARITY Decide whether the quadrilaterals are similar. Explain your reasoning.



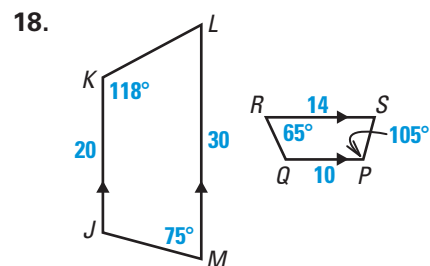
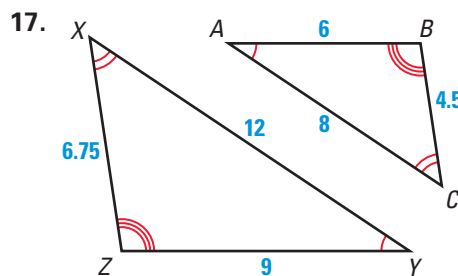
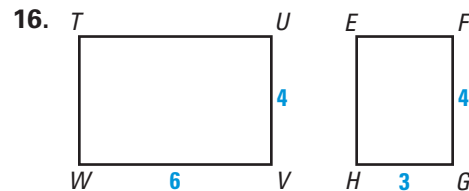
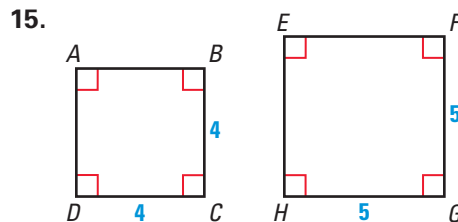
11. $ABCD$ and $FGHE$

12. $ABCD$ and $JKLM$

13. $ABCD$ and $PQRS$

14. $JKLM$ and $PQRS$

DETERMINING SIMILARITY Decide whether the polygons are similar. If so, write a similarity statement.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 8–10

Example 2: Exs. 11–18

Example 3: Exs. 19–30,
43, 44

Example 4: Exs. 19–30,
46–48

Example 5: Exs. 39–42

USING SIMILAR POLYGONS $PQRS \sim JKLM$.

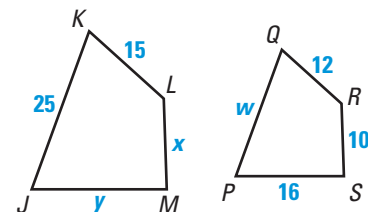
19. Find the scale factor of $PQRS$ to $JKLM$.

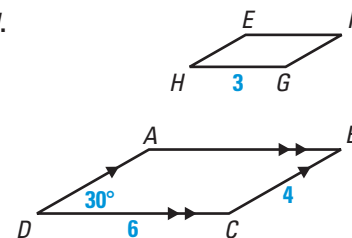
20. Find the scale factor of $JKLM$ to $PQRS$.

21. Find the values of w , x , and y .

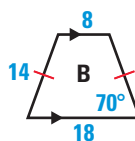
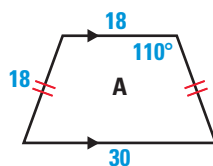
22. Find the perimeter of each polygon.

23. Find the ratio of the perimeter of $PQRS$ to the perimeter of $JKLM$.

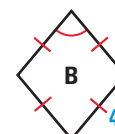
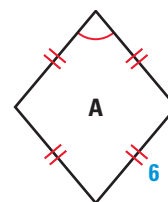


USING SIMILAR POLYGONS $\square ABCD \sim \square EFGH$.24. Find the scale factor of $\square ABCD$ to $\square EFGH$.25. Find the length of \overline{EH} .26. Find the measure of $\angle G$.27. Find the perimeter of $\square EFGH$.28. Find the ratio of the perimeter of $\square EFGH$ to the perimeter of $\square ABCD$.**DETERMINING SIMILARITY** Decide whether the polygons are similar. If so, find the scale factor of Figure A to Figure B.

29.



30.

**STUDENT HELP****HOMEWORK HELP**

Visit our Web site
www.mcdougallittell.com
 for help with problem
 solving in Exs. 31–38.

LOGICAL REASONING Tell whether the polygons are *always*, *sometimes*, or *never* similar.

31. Two isosceles triangles

32. Two regular polygons

33. Two isosceles trapezoids

34. Two rhombuses

35. Two squares

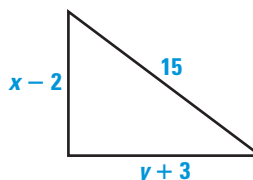
36. An isosceles and a scalene triangle

37. Two equilateral triangles

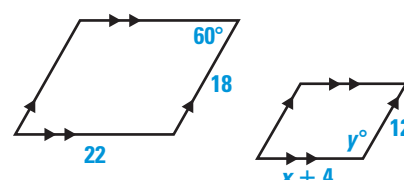
38. A right and an isosceles triangle

USING ALGEBRA The two polygons are similar. Find the values of x and y .

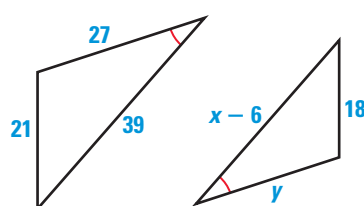
39.



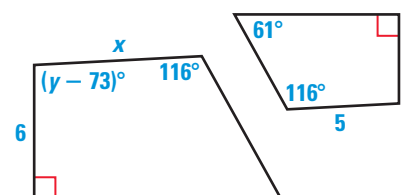
40.



41.



42.

**FOCUS ON APPLICATIONS****DIGITAL TELEVISION**

screens contain over 6 times
 as many pixels (the tiny dots
 that make up the picture) as
 standard analog screens.

TV SCREENS In Exercises 43 and 44, use the following information.

Television screen sizes are based on the length of the diagonal of the screen. The *aspect ratio* refers to the length to width ratio of the screen. A standard 27 inch analog television screen has an aspect ratio of 4:3. A 27 inch digital television screen has an aspect ratio of 16:9.

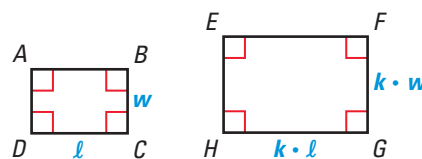
43. Make a scale drawing of each television screen. Use proportions and the Pythagorean Theorem to calculate the lengths and widths of the screens in inches.

44. Are the television screens similar? Explain.

45. **PROOF** Prove Theorem 8.1 for two similar rectangles.

GIVEN $\triangleright ABCD \sim EFGH$

PROVE $\triangleright \frac{\text{perimeter of } ABCD}{\text{perimeter of } EFGH} = \frac{AB}{EF}$



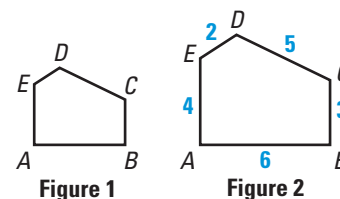
46. **SCALE** The ratio of the perimeter of $WXYZ$ to the perimeter of $QRST$ is $7.5:2$. Find the scale factor of $QRST$ to $WXYZ$.
47. **SCALE** The ratio of one side of $\triangle CDE$ to the corresponding side of similar $\triangle FGH$ is $2:5$. The perimeter of $\triangle FGH$ is 28 inches. Find the perimeter of $\triangle CDE$.
48. **SCALE** The perimeter of $\square PQRS$ is 94 centimeters. The perimeter of $\square JKLM$ is 18.8 centimeters, and $\square JKLM \sim \square PQRS$. The lengths of the sides of $\square PQRS$ are 15 centimeters and 32 centimeters. Find the scale factor of $\square PQRS$ to $\square JKLM$, and the lengths of the sides of $\square JKLM$.

Test Preparation

49. **MULTI-STEP PROBLEM** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is $7:10$.

a. Copy and complete the table.

	AB	BC	CD	DE	EA
Figure 1	?	?	?	?	?
Figure 2	6.0	3.0	5.0	2.0	4.0



b. Graph the data in the table. Let x represent the length of a side in Figure 1 and let y represent the length of the corresponding side in Figure 2. Determine an equation that relates x and y .

c. **ANALYZING DATA** The equation you obtained in part (b) should be linear. What is its slope? How does its slope compare to the scale factor?

★ Challenge

TOTAL ECLIPSE Use the following information in Exercises 50–52.

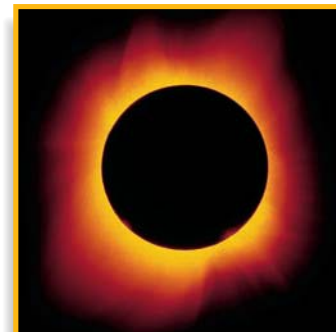
From your perspective on Earth during a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The ratio of the radius of the moon to its distance to Earth is about the same as the ratio of the radius of the sun to its distance to Earth.

Distance between Earth and the moon: 240,000 miles

Distance between Earth and the sun: 93,000,000 miles

Radius of the sun: 432,500 miles

50. Make a sketch of Earth, the moon, and the sun during a total eclipse of the sun. Include the given distances in your sketch.
51. Your sketch should contain some similar triangles. Use the similar triangles in your sketch to explain a total eclipse of the sun.
52. Write a statement of proportionality for the similar triangles. Then use the given distances to estimate the radius of the moon.



EXTRA CHALLENGE

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MIXED REVIEW

FINDING SLOPE Find the slope of the line that passes through the given points. (Review 3.6 for 8.4)

53. $A(-1, 4), B(3, 8)$

54. $P(0, -7), Q(-6, -3)$

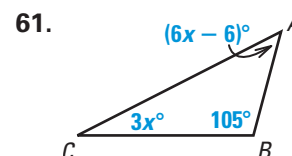
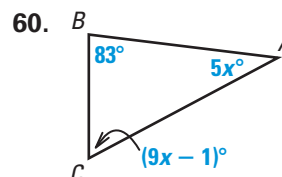
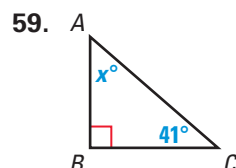
55. $J(9, 4), K(2, 5)$

56. $L(-2, -3), M(1, 10)$

57. $S(-4, 5), T(2, -2)$

58. $Y(-1, 6), Z(5, -5)$

FINDING ANGLE MEASURES Find the value of x . (Review 4.1 for 8.4)



SOLVING PROPORTIONS Solve the proportion. (Review 8.1)

62. $\frac{x}{9} = \frac{6}{27}$

63. $\frac{4}{y} = \frac{2}{19}$

64. $\frac{5}{24} = \frac{25}{z}$

65. $\frac{4}{13} = \frac{b}{8}$

66. $\frac{11}{x+2} = \frac{9}{x}$

67. $\frac{3x+7}{5} = \frac{4x}{6}$

QUIZ 1

Self-Test for Lessons 8.1–8.3

Solve the proportions. (Lesson 8.1)

1. $\frac{p}{15} = \frac{2}{3}$

2. $\frac{5}{7} = \frac{20}{d}$

3. $\frac{4}{2x-6} = \frac{16}{x}$

Find the geometric mean of the two numbers. (Lesson 8.2)

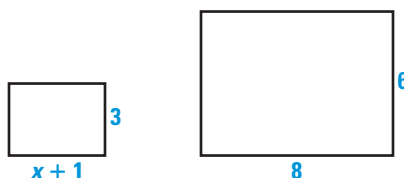
4. 7 and 63

5. 5 and 11

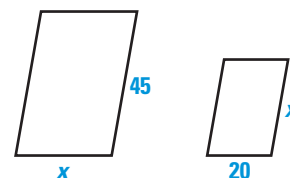
6. 10 and 7

In Exercises 7 and 8, the two polygons are similar. Find the value of x . Then find the scale factor and the ratio of the perimeters. (Lesson 8.3)

7.



8.



 **COMPARING PHOTO SIZES** Use the following information. (Lesson 8.3)

You are ordering your school pictures. You decide to order one 8×10 (8 inches by 10 inches), two 5×7 's (5 inches by 7 inches), and 24 wallets ($2\frac{1}{4}$ inches by $3\frac{1}{4}$ inches).

9. Are any of these sizes similar to each other?

10. Suppose you want the wallet photos to be similar to the 8×10 photo. If the wallet photo were $2\frac{1}{2}$ inches wide, how tall would it be?